Covariance Analysis of Business Student's Mathematical Performance Model: A Case of Khon Kaen University International College

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Abstract

This research paper places an emphasis on the association between mathematical skills and the qualifications of students whose mathematics related courses were taught in English. Business students in Khon Kaen University International College were used as samples. This study tries to determine the qualifications which may affect the academic performance of students in math related courses. The study employed Log-linear multiple regression as a tool to represent this relationship. The significance of each factor was then tested using the Lagrange Multiplier Test (LM test). The results yielded from Covariance Analysis, suggested that the student's ages significantly demonstrate a negative correlations between older students and lower scores. As they aged their scores decreased. On one hand, Sciences/math subject area from high school interacts significantly with both student's age and their English skill – negatively for age and positively for English skill. Moreover, the three nationalities of students – Thai, Chinese, and Vietnamese – interact significantly with student's ages and moves in the same direction. Lastly, quadratic relationships were expressed on both students' ages and their IQs, implying that as student's ages and IQs increase, their academic performances in mathematics related courses increase with increasing rates.

Keywords: Covariance Analysis, LM Test, Mathematical Skills, Log-linear Multiple Regression

Introduction

Khon Kaen University International College (KKUIC) offers many mathematics related courses such as: Business Finance, Accounting for Management, Business Mathematics and Statistics, and Taxation. Regarding the nature of international programs, the instructors of those courses use English as a medium of instruction. Each semester, the total number of students taking the courses is more than 200. Due to differences in the academic backgrounds of students, their performances in those courses tend to vary. In other words, the difference in mathematical skills of students should be explained by their diverse qualifications. Regarding the literatures by many scholars and the researcher's observation, the variables influencing mathematical skills (for courses taught in English) are: student's age (Bull, 2014), IQ (Alloway, 2011), English skill (Pyburn, 2014), in-class participation (Webb, 2014), number of prerequisite classes taken, subject areas from high school, genders, and nationalities. In turns, this study hypothesized that the above mentioned variables affected college student's mathematical skills in different ways.

In the study of executive functioning, Bull (2014) stated that as age increases, there exists a greater emphasis on the children's development of arithmetical skills, which form the basis for being able to complete multi-step word problems, algebraic word problems, and other curriculum based tests. Additionally, Alloway (2011) investigated the contribution of working memory (signifying child's IQ) and verbal ability (measured by vocabulary) to mathematical skills in children. The study concluded that memory skills uniquely predicted mathematical skills and arithmetical abilities of children. Another essential factor affecting student's mathematical performance is the language skills. Pyburn (2014) tried to assess the relation between language comprehension and performance in general chemistry class. The finding proposed that language was fundamental to the teaching and learning of chemistry, as it is to all sciences. In KKUIC, about 90 percent of students are non-native English speakers. With all the classes taught in English, students with sufficient English foundation are expected to perform better academically. Moreover, Webb (2014) pointed out the importance of student's participation in her study. The results indicated that the level of student engagement with each other's ideas and the incidence of students providing detailed explanations of their problem-solving strategies were positively related to student achievement. Besides the factors previously studied by the above scholars, the researcher added more suspicious variables that might affect the student's mathematical performances namely: number of prerequisite classes taken, subject areas from high school, genders, and nationalities. The descriptions of these variables are provided in the methodology part below.

In sum, "Covariance Analysis of Business Student's Mathematical Performance Model:A Case of Khon Kaen University International College" was conducted in order to study various variables which may impact mathematical skills of students whose courses were taught in English. In contrast to the empirical studies by both Bull (2014) and Alloway (2011), this study used international program undergraduate students aging around 17 to 23 as samples. The model was formulated by log-linear multiple regression (Fienberg, 2010). To explain the relationship among variables in a more meaningful way, besides regular dummy and quantitative variables, the data was transformed into interaction terms (Cox, 1984) and squared terms (Stigler, 1974) (Gergonne, 1974). Then the model was tested using the Lagrange Multiplier Testing (LM testing) (Engle, 1983), ((Bera, 2001). Lastly the marginal effects of each determinant were interpreted by Covariance Analysis.

1. Objectives of Study

To identify the relationship between mathematical skills and qualifications of students whose mathematics related courses were taught in English

2. Methodology

2.1 Tools to be Employed

Covariance Analysis of Log-linear Multiple Regression Model using Quantitative and Dummy Independent Variables (Ramanathan, 2002):

a) The model is estimated by:

$$100\left(\frac{\hat{Y}_1}{\hat{Y}_0}-1\right) = 100\{exp[\hat{\beta}_3 - \frac{1}{2}\widehat{Var}(\hat{\beta}_3)]-1\}, \quad (1)$$

Where $\hat{Y}_0$ and $\hat{Y}_1$ are the dependent variables when D_1 (dummy) = 1 and D_o = 0 respectively; *exp* is exponential function; and $\widehat{Var}$ is the estimated variance. If the model has an interactive term so that it becomes:

$$ln(Y) = \beta_1 + \beta_2 X + \beta_3 D + \beta_4 DX + u \quad (2)$$

the corresponding expression is much more complicated. Verify that, in this case, it is:

$$100\left(\frac{\hat{Y}_1}{\hat{Y}_0}-1\right) = 100\{exp[\hat{\beta}_3 - \frac{1}{2}\widehat{Var}(\hat{\beta}_3 + \hat{\beta}_3 X)]-1\} \quad (3)$$

b) The variance expression depends on the value of *X* and also involves a linear combination of random variables. As for marginal effects of each determinant, the marginal effect of quantitative variable is

$\frac{\delta ln(\hat{Y})}{\delta X} = \hat{\beta}_3 + \hat{\beta}_3 D.$ Thus, using logarithmic differentiation property, it becomes:

$$100\frac{\Delta Y}{Y} = 100(\beta_2 + \beta_4 D)\Delta X \quad (4)$$

It follows that $100\hat{\beta}_2$ is the approximate percent of change in *Y* for a unit of change in *X* when *D = 0* and that $100(\hat{\beta}_2 + \hat{\beta}_4)$ is the approximate percent of change in *Y* for a unit change in *X* when *D = 1.*

2.2 Implementation

a) Cross-sectioned data presented below was collected from 107 students from KKUIC who took math-related courses in 2013:

MSKILL = mathematical skills, represented by student's raw scores on math related courses (e.g. Business Mathematics and Statistics; Business Finance; Accounting for Management; and Taxation).

Data Collection: 107 students from Business Finance, Accounting for Management, Business Mathematics and Statistics, and Taxation classes in academic year 2013 were asked to complete the basic mathematics test designed by the researcher. This test covered basic mathematics knowledge necessary for undergraduate business students. A time limit of 120 minutes was allowed for the test. The contents of the test included the following topics: 1) Whole Numbers, Fractions, and Decimals, 2) Percentage and Ratios, 3) Powers and Roots, 4) Basic Algebra, 5) Solving Equations, and 6) Basic Time Value of Money**.** The test consisted of 100 points. The students' scores were grouped into different grades: A = 80 to 100, B+ = 75 to 79.99, B = 70 to 74.99, C+ = 65 to 69.99, C = 60 to 64.99, D+ = 55 to 59.99, D = 50 to 54.99, and F = below 50. Then the grades were transformed into the following scores: A = 4, B+ = 3.5, B = 3, C+ = 2.5, C = 2, D+ = 1.5, and D = 1.

AGE = student's ages (Bull, 2014).

IQ = student's IQs measured by IQ scores (Alloway, 2011).

Data Collection: The same students were also asked to complete IQ Test 1 of *"Advanced IQ Tests"*(Carter, 2014). The test consisted 30 multiple-choice and fill-in-the-blank questions. A time limit of 120 minutes was allowed for the test. A piece of A4 paper was provided for the students to take notes on calculation. The use of calculator was not permitted due to the fact that the numerical questions were designed to test the students' power of mental arithmetic and their aptitude when working with numbers. After the tests were graded, the students' raw scores (out of 30) were converted to the total of 12 and the decimals were rounded off.

ATTEN = student's attention to classes measured by student's participation scores (Webb, 2014)

Data Collection: The students' class attendants were checked by the instructors of the above classes. In KKUIC, one course consists of 45 credit hours and is divided into 15 classes. When a student attended the class, one point was given for his/her participation. Therefore, the total participation score was 15 points. After the course was over, the participation score of 15 was then converted to the total of 10.

ESKILL = English skill (measured by the average of student's grades on Academic English course and Critical Reading and Writing course) (Pyburn, 2014).

Data Collection: KKUIC offers the following English courses as parts of communication skills in general education. The courses are English for Communication in Multicultural Societies, English for Lifelong Learning, Academic English, and Critical Reading and Writing. However, the researcher chose to use just the student's average grade of only 2 courses (Academic English and Critical Reading and Writing) because these 2 courses were sufficient enough to represent the student's overall English skills which are reading, writing, listening, and speaking. The student's grades were in the following scales: A = 4, B+ = 3.5, B = 3, C+ = 2.5, C = 2, D+ = 1.5, and D = 1.

PREREQ = student's number of prerequisite math-related courses taken (researcher's observation).

Data Collection: The researcher basically counted the number of prerequisite math-related courses that each student took before he/she took the basic mathematics test designed by the researcher. Usually, students with higher years of study tend to complete more number of prerequisites as compared to those with lower years.

GEN = student's gender (male = 1 and female = 0) (self-observation).

SC_M = students taking sciences/math subject area from high school (=1, others = 0) (self-observation).

A_M = students taking arts/math subject area from high school (=1, others = 0) (self-observation).

A = students taking pure arts subject area from high school (=1, others = 0) (self-observation).

Data Description: The 3 main subject areas for high school education in Thailand are: Sciences/Mathematics (SC_M), Arts/Mathematics (A_M), and Pure Arts (A). Business programs in KKUIC accept students from various backgrounds as mentioned above. The majority of students in the programs are from the sciences/math subject area, accounting for 60%. About 30% are from arts/math and 10% are from pure arts.

THA = Thai students (=1, others = 0) (self-observation).

VIET = Vietnamese students (=1, others = 0) (self-observation).

CHI = Chinese students (=1, others = 0) (self-observation).

OTHER = students with other nationalities than THA, VIET, and CHI (=0), used as control group

Data Description: As for the nationalities of students taking those math-related courses, Thai (THA), Vietnamese (VIET), and Chinese (CHI) were considered the majority. Of this, about 80% were Thai. Only a few students were westerners and English native speakers, which were counted as OTHER.

b) The Lagrange Multiplier Test was used to determine whether some or all stated student characteristics are significant. The following list includes a number of quantitative variables, dummy variables, interactions terms, and squared variables generated through transformations:

Quantitative Variables

0)const 1)MSKILL 2)AGE 3)IQ 4)ATTEN 5)ESKILL 6)PREREQ

Note: "const" refers to a constant term.

Dummy Variables (Suits, 1957)

7)GEN 8)SC_M 9)A_M 10)A 11)THA 12)VIET 13)CHI 14)OTHER

Interaction Terms (Cox, 1984)

An interaction term is a multiplication of quantitative and dummy variables. To develop the terms, a dummy variable was paired up and multiplied with each quantitative variable at a time starting from the first dummy variable to the last one (from GEN to OTHER):

15)GENxAGE 16)GENxIQ 17)GENxATTEN 18)GENxESKILL 19)GENxPREREQ 20) SC_MxAGE 21)SC_MxIQ 22)SC_MxATTEN 23)SC_MxESKILL 24)SC_MxPREREQ 25)A_MxAGE 26)A_MxIQ 27)A_MxATTEN 28)A_MxESKILL 29)A_MxPREREQ 30) THxAGE 31)THxIQ 32)THxATTEN 33)THxESKILL 34)THxPREREQ 35)VxAGE 36) VxIQ 37)VxATTEN 38)VxESKILL 39)VxPREREQ 40)CxAGE 41)CxIQ 42)CxATTEN 43)CxESKILL 44)CxPREREQ 45)OxAGE 46)OxIQ 47)OxATTEN 48)OxESKILL 49) OxPREREQ

Note: "O" in 45) to 49) refers to OTHER.

Squared Variables (Stigler, 1974) (Gergonne, 1974)

A squared term was derived from a quantitative variable multiplied by the value of itself. The square terms help capture the quadratic trends of the suspected variables:

50)sq_AGE 51)sq_IQ 52)sq_ATTEN 53)sq_ESKILL 54)sq_PREREQ

c) Using Gretl software, ln(MSKILL) was regressed against a constant, AGE, IQ, ATTEN, ESKILL, and PREREQ, and then the residuals was saved and named uhat1 (Ramanathan, 2002). The following is the auxiliary regression that regresses the residuals against all the variables in the unrestricted model:

Table 1

OLS, using observations 1-107, Dependent variable: uhat 1

Table 1

OLS, using observations 1-107, Dependent variable: uhat1

	Coefficient	Std. Error	t-ratio	p-value	
const	30.2235	15.6263	1.9341	0.05732	*
AGE (Age)	-1.44041	0.731539	-1.969	0.05309	*
IQ (IQ)	-0.0327548	0.193424	-0.1693	0.86604	
ATTEN (Attendance)	-2.2545	2.26967	-0.9933	0.32413	
ESKILL (English Skills)	0.0358782	0.474836	0.0756	0.94	
PREREQ (Prerequisites)	-0.133772	0.187374	-0.7139	0.47775	
GEN (Gender)	-5.74171	3.10763	-1.8476	0.06907	*
SC_M (Sciences/Math)	-4.65232	2.48461	-1.8725	0.06551	*
A_M (Arts/Math)	-5.47474	2.784	-1.9665	0.05338	*
THA (Thai)	-4.56046	7.98954	-0.5708	0.57004	
VIET (Vietnamese)	-13.1762	19.899	-0.6622	0.51014	
CHI (Chinese)	-5.54755	6.98792	-0.7939	0.43007	
GENxAGE (Interaction Terms)	0.162422	0.0932886	1.7411	0.08626	*
GENxIQ (Interaction Terms)	-0.0193638	0.0377294	-0.5132	0.60948	

GENxATTEN (Interaction Terms)	0.308841	0.215603	1.4325	0.15666	
GENxESKILL (Interaction Terms)	-0.123716	0.112508	-1.0996	0.27543	
GENxPREREQ(Interaction Terms)	-0.0513443	0.0650067	-0.7898	0.43241	
SC_MxAGE (Interaction Terms)	0.105963	0.0836507	1.2667	0.20964	
SC_MxIQ (Interaction Terms)	0.00170322	0.0371742	0.0458	0.96359	
SC_MxATTEN (Interaction Terms)	0.237241	0.157144	1.5097	0.13582	
SC_MxESKILL (Interaction Terms)	0.127448	0.120828	1.0548	0.29531	
SC_MxPREREQ (Interaction Terms)	-0.0241213	0.068717	-0.351	0.72667	
A_MxAGE (Interaction Terms)	0.0464564	0.0992389	0.4681	0.64121	
A_MxIQ (Interaction Terms)	0.0312891	0.0492229	0.6357	0.52716	
A_MxATTEN (Interaction Terms)	0.471104	0.17502	2.6917	0.00897	***
A_MxESKILL (Interaction Terms)	-0.0439055	0.135549	-0.3239	0.74702	
A_MxPREREQ (Interaction Terms)	-0.0470596	0.0768783	-0.6121	0.54252	
THxAGE (Interaction Terms)	0.251002	0.265314	0.9461	0.34752	
THxIQ (Interaction Terms)	-0.0538028	0.141382	-0.3805	0.70474	
THxATTEN (Interaction Terms)	1.43E-05	0.557737	0	0.99998	
THxESKILL (Interaction Terms)	0.0450185	0.374379	0.1202	0.90465	
THxPREREQ (Interaction Terms)	0.192022	0.137446	1.3971	0.167	
VxAGE (Interaction Terms)	0.51268	0.605755	0.8463	0.40037	
VxIQ (Interaction Terms)	0.311118	0.729132	0.4267	0.67097	
CxAGE (Interaction Terms)	0.304372	0.282076	1.079	0.28444	
sq_AGE (Square Age)	0.0257414	0.0122703	2.0979	0.03969	**
sq_IQ (Square IQ)	0.0048721	0.00584979	0.8329	0.40788	
sq_ATTEN (Square Attendance)	0.103553	0.115139	0.8994	0.37168	
sq_ESKILL (Square English Skills)	-0.0189182	0.04052	-0.4669	0.6421	
sq_PREREQ (Square Prerequisites)	0.00733153	0.0244114	0.3003	0.76485	
Mean dependent var	0		S.D. dependent var	0.23146	
Sum squared resid	3.273817		S.E. of regression	0.22105	
R-squared	0.423506		Adjusted R-squared	0.087935	
F(39, 67)	1.262045		P-value(F)	0.198839	
Log-likelihood	34.72124		Akaike criterion	10.55753	
Schwarz criterion	117.4707		Hannan-Quinn	53.8987	

d) The next step was to select variables to be added to the basic model using simple but arbitrary rule of thumb of including newly added variables that have p-value less than 0.05.

e) Then we regressed selected variables with ln(MSKILL) and omitted variables with insignificant coefficients**,** a few at a time, until all coefficients were significant at 5 percent. (The results are shown in Results section below)

f) Finally, the complete model was estimated with all square and interaction terms.

3. Results

Table 2

OLS, using observations 1-107, Dependent variable: l_MSKILL

Table 2 indicates the results of Lagrange Multiplier Test on all the sus-

	Coefficient	Std. Error	t-ratio	p-value	
const	10.5549	1.748	6.0383	<0.00001	***
AGE	-0.650014	0.170765	-3.8065	0.00025	***
SC_MxAGE	-0.0237233	0.0054185	-4.3782	0.00003	***
SC_MxESKILL	0.171011	0.0337917	5.0607	<0.00001	***
THxAGE	0.0289435	0.0108876	2.6584	0.00917	***
VxAGE	0.0370959	0.0122071	3.0389	0.00304	***
CxAGE	0.0281024	0.0113152	2.4836	0.0147	**
sq_AGE	0.0146108	0.00384282	3.8021	0.00025	***
sq_IQ	0.00175699	0.000714789	2.4581	0.01572	**

Mean dependent var	4.135463	S.D. dependent var	0.261524
Sum squared resid	4.3389	S.E. of regression	0.210415
R-squared	0.401516	Adjusted R-squared	0.352661
F(8, 98)	8.218396	P-value(F)	1.86E-08
Log-likelihood	19.65221	Akaike criterion	-21.30441
Schwarz criterion	2.751048	Hannan-Quinn	-11.55265

pected qualifications (variables) of mathematics students in KKUIC. The value R-squared statistic of 0.40 is quite acceptable for cross-sectioned data with the corresponding p-value (F) of 1.86E-08. Additionally, p-values of all the selected variables (AGE, SC_MxAGE, SC_MxESKILL, THxAGE, VxAGE, CxAGE, sp_AGE, sq_IQ) demonstrate that all coefficients in the model are significant at 5 percent or lower level. As a result, the logarithm of MSKILL is well explained by the constructed model, and hence, clearly signifying the superiority of this model. Moreover, the results neglect some variables namely ATTEN, PREREQ, GEN, A, and OTHER, suggesting that they are insignificant.

4. Conclusion and Discussion

To analyze the impact of each variable on MSKILL in-depth, the marginal effects of each determinant was examined separately:

Age: Student's age is essential in explaining their mathematical skills. The model explained its findings differently from Bull's study (Bull, 2014). Student's age exhibits significant nonlinearity and also interacts significantly with sciences/math subject area and student's nationalities (Thai, Vietnamese, and Chinese). The partial effect is:

$$\frac{\Delta \ln(\widehat{MSKILL})}{\Delta AGE} = -0.65 + 0.29AGE - 0.024SC\_M + 0.029TH + 0.037V + 0.028C \quad (5)$$

According to the above derivative, the additional raw score on mathematics related courses for one additional year of age increases with an increasing rate as students get older. Interestingly, one year older in age indicates 2.4 percent decrease in student's scores on mathematics related courses for students from sciences/math high school subject area. This is because the majority of students taking mathematics related courses are from sciences/math subject area in high school As their ages increase, students are moving towards the upper division courses with increasingly higher degree of difficulty, resulting in lower scores. In addition, student's nationalities interact significantly with their ages. On average, one year older in age means 2.9 percent increase in scores for Thai students, 3.7 percent increase in scores for Vietnamese students, and 2.8 percent increase in scores for Chinese students.

English Skill: The result from the model agreed with Pyburn's study (Pyburn, 2014) but was presented in a more meaningful manner. English skills of students significantly interact with students who took science/math subject area from high school level. The partial effect is:

$$\frac{\Delta \ln(\widehat{MSKILL})}{\Delta ESKILL} = 0.17SC\_M \quad (6)$$

Regarding the above derivative, as the average of student's grades on Academic English class and Critical Reading and Writing class increase by one point, students from science and math subject area can score 17 percent more.

IQ: The model suggested a direct relationship between IQ and student's mathematical performance, which is in line with Alloway's findings (Alloway, 2011). However, student's IQ represents nonlinear relationship with student's mathematical skills at the same time. The partial effect is:

$$\frac{\Delta \ln(\widehat{MSKILL})}{\Delta IQ} = 0.0035IQ \quad (7)$$

As IQ scores increase by one point, student's additional raw score in mathematics related courses increases by 0.35 percent along with IQ. In other words, as student's IQ rises by one point, student's mathematical skills will increase with an increasing rate.

Nationalities: The sample of students belonged to 4 different nationalities: Thai, Vietnamese, Chinese, and others, where other nationalities are used as the control group. The estimated partial effects for log MSKILL for each of the non-control group are:

Thai: *0.029AGE*

Vietnamese: *0.037AGE*

Chinese: *0.028AGE*

As pointed out earlier, age significantly has a positive effect on Thai, Vietnamese, and Chinese students but not for students with other nationalities.

Sciences/Math Subject Area: The partial effect for log MSKILL is:

$$\frac{\Delta \ln(\widehat{MSKILL})}{\Delta SC\_M} = -0.24AGE + 0.17ESKILL \quad (8)$$

As indicated earlier, students with science/math subject area from high school have the raw scores in mathematics related courses that negatively interact with age but positively interact with English skill.

5. Recommendations

Recommendations from the analysis of the Lagrange Multiplier can be categorized into three main groups:

a) In order to effectively select new students, younger students who have completed sciences/math subjects are more preferable. This is because sciences/math are the only subject areas which seem to impact students academic performances on mathematics related courses. Moreover, Thai, Chinese, and Vietnamese nationalities were selected for mathematics related courses because the model significantly demonstrated positive impacts of these nationalities on student's academic performances. Additionally, a student's IQ also resulted in a somewhat positive relationship with their mathematical proficiency. Thus, it is recommended that an IQ test be included in the admission process.

b) For effective teaching strategy and student's learning success, it is recommended that students from sciences/math subject areas in high school take English courses (either provided by the institution itself or outside institution) along with their regular courses. This is because the model indicates that English skill have the highest value of coefficient (17 percent) compared with other variables and move in the same direction as the academic performance of students. Further taking its p-value into account (<0.0001), English skill becomes the most important factor leading students to do well in mathematics related courses taught in English.

c) As for possible future studies, in order to increase the value of R^2 (the goodness of fit) which may result in a greater accuracy of the model, changes in statistical model and the addition of new variables is strongly recommended.

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