Tolerance Analysis of Pivot ball bearing in Hard Disk Drive

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B. Prannattee*, K. Tangchaichit

Abstract

This paper explains the tolerance study of a pivot ball bearing of the actuator–arm mechanism in the hard disk drive (HDD), which can be generated a mathematical modeling equation. The mathematical modeling equation can describe the relation of the individual part dimensions which affects the clearance or interference. Moreover, it can used for model developments, assembly problem analysis and complete assembly predictions. This study is based on the worst case analysis and statistical analysis which provide the maximum clearance and minimum clearance expressed in the Min/Max Chart. With the common tolerance analysis methods, the Min/Max Charts are combined with the Parametric Simulation to generate the mathematical modeling equation by linear regression analysis. The result of the worst case analysis differs from the mathematical modeling equation -0.87% in nominal gap, 9.72% in maximum gap, and -18.60% in minimum gap. In addition, the results of statistical analysis differ from mathematical modeling equation -0.87% in nominal gap, 2.26% in maximum gap and -6.25% in minimum gap.

Keywords: Tolerance, pivot ball bearing, mathematical modeling equation, clearance, interference, worst case analysis, statistical analysis, Min/max Charts, Parametric simulation, gap.

Introduction

Recently, the frictional effects in a pivot ball bearing in HDD have grown to be one of the active research topics for both servo and actuator design in the storage business. Early HDD used the stepper motor actuator systems which were slow, high temperature, position sensitive, and were generally unreliable (Blount, 2001). Presently, most HDD servo system uses a voice coil motor (VCM) actuator to actuate the read/write (R/W) recording arm assembly.

However, the friction and nonlinearities results in a large residual error and deteriorate the performance of head positioning of HDD servo system and the other mechanical servo systems.

The factors are important to the nonlinear response from friction in the actuator rotary pivot bearing and data flex cable in the VCM actuator(Chang et al., 1999; Takaishi et al., 1996; Peng et al., 2003; Peng et al., 2003; Peng et al., 2005.). The main factor of nonlinear
response depends on the ball bearing friction (Takaishi et al., 1996). One of the answers to nonlinear is presented as a method to reduce nonlinearities by using relay function in 3.5-inch HDD which has the uncertainty result from Laser Doppler Vibrometer (LDV), drift occurring in the experiment (Chang et al., 1999). The other method (Peng et al., 2005), modeling and compensation of nonlinearities and friction in micro HDD system by using nonlinear feed back control, composite nonlinear feedback (CNF) and proportional integral derivative (PID), to control in settling time by 76%.

In most of the pivot studies on HDD, the bearing is focused at nonlinearities from friction and the feedback control is investigated. A few interesting researches of the ball bearing were about the results from the structure design that affect the overall performances. The finite element analysis (FEA) simulation of the pivot bearing is presented (Luo et al., 2006) by the stiffness calculation where the results between theoretical and FEA by the use of software are similar. A new design is proposed (He et al., 2003) where the 4-leaf cross-flexural pivot can replace the bearing currently used ball bearing in HDD when compared the performances, the results is to approach but a novel design is the prototype and its application for developing in the future.

In previous research, tolerance analysis of pivot bearing in HDD is a novel approach. The basic principles of tolerance analysis are based on worst case analysis and statistical analysis (Shah et al., 2007; Lee and Yi, 1997; Turner, 1998; Katsumaru et al., 2005; Ceglarek and Shi, 2003; Li and Roy, 2001; Wang, 2002; Lee and Yi, 1994; Hu and Gu, 1994.) for using in the assembled parts. In 1991 (Schlatter, 1996) tolerance analysis were first developed in 5.25-inch HDD by using computer aided 3D, the results are known to predicted run-out, disk-stack envelop, and imbalance number. Because of this, tolerance analysis is one of the most methods that using the developed the pivot ball bearing model.

This paper focuses at the tolerance analysis of the pivot ball bearing in HDD based on tolerance stack-up analysis (Meadows, 2001) which can be referred to the ANSI Y14.5M (ANSI Y145M, 1982). The key concept is to propose an approach presenting a relationship of individual parts that affect the clearance by using mathematical modeling equation.

**Basic Principles**

Tolerance is defined as the total amount by which a specific dimension is permitted to vary. Therefore, geometric tolerance is the general term applied to the category of tolerances used to control form, profile, orientation, location, and run-out. Basic principles of tolerance analysis are 1-D Min/ max charts and parametric simulation. The methods can be shown

3.1 Worst Case Analysis (Shah et al., 2007) is the method of analyzing a piece of a design using the high and low end of the tolerance spread for each parameter, which is used whenever the critical parameters within a design and the effects of these parameters in worst case conditions are need to be evaluated.

If \( A = \pm d_1 \pm d_2 \pm d_3 \pm \ldots \) \( i = 1, 2, 3, \ldots, n \)

then mean value \( A \) can be written as

\[
\bar{A} = \pm \bar{d}_1 \pm \bar{d}_2 \pm \bar{d}_3 \pm \ldots = \sum_{i=1}^{n} \bar{d}_i
\]
and
\[ \Delta A = A_{\text{max}} - A_{\text{min}} = 2 \times (\Delta d_1 + \Delta d_2 + \Delta d_3 + \ldots) \]  
\[ = 2 \times \sum_{i=1}^{n} \Delta d_i \]  

3.2 Statistical analysis (Shah et al., 2007) typically assumes that the contributors \( d_i \) are all normally distributed and their nominal values are set at the mean, and the equal bilateral tolerances are set at \( n \ast \sigma \) (\( n \) times the standard deviation, typically 3). Therefore, the mean and standard deviation of \( A \) can be found from
\[ A = \pm \bar{d}_i \pm \bar{d}_i = \sum_{i=1}^{n} (\text{sign})_i d_i \]  
\[ \sigma^2 = \sigma_1^2 + \sigma_2^2 = \sum \left( \frac{\Delta d_i}{n} \right)^2 \]  

where

\( A \) is an arithmetic sum or difference of the contributing size dimensions \( d_i \)

\( \Delta d_i \) are bilateral tolerances for contributor \( i \) about mean value \( \bar{d}_i \)

Assume that the acceptable values of the parameters lie between upper limit and lower limit (see Figure 1.), then the shaded area under the curve, which can be found from statistical Tables, will give the Acceptance Rate. (% parts acceptable)

3.3 The common tolerance analysis method combines the Min/Max Chart and Parametric simulation which can handle both dimensional and geometric tolerances. In the Parametric simulation method, random numbers are first generated and values of \( d_1, d_2, d_3, \ldots d_n \) are selected from pre-assigned values in Tables which reflect the multipliers. (i.e. Geometric Multiplier, Statistic Multiplier, and Probability Multiplier) Assume that the acceptable values of the parameters lie between upper limit and lower limit see the examples in Figure 5–8.

3.4 Linear regression analysis is a statistical tool for the investigation of relationships between variables to determine the values of parameters for a function that best fits a set of data observations provided. Linear regression is a regression method that models the relationship between a dependent variable \( Y \) independent variables \( X_i, i = 1, 2, 3, \ldots n \) and a random error term \( \varepsilon \). The model can be written as
\[ Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_n X_n + \varepsilon \]  

where
\( \beta_0 \) is the \( Y \) - intercept. (Constant term)
\( \beta_i \) is the respective parameters of independent variables. (Regression coefficient)
\( n \) is the number of parameters to be estimated in the linear regression.

The relation of the response (the dependent variable \( Y \)) to the independent variables is assumed to be a linear function of the parameters.

Tolerance Analysis Methodology

The tolerance analysis starts upon starting to design a pivot ball bearing and ends when the model
is established. Accurate tolerance analysis methodology depends on the physical significance of the model. The following section describes tolerance analysis methodology of a pivot ball bearing.

4.1 To study the construction of a pivot ball bearing actuator-arm mechanism in 3.5-inch hard disk drive/7200 rpm/ 2 R/W heads.

Figure 2. Mechanical structure of the actuator in HDD.

Figure 3. The structure of a pivot ball bearing in HDD.

4.2 To study the tolerance stack-up analysis including to dimensional and geometric tolerance of a pivot ball bearing and specify all parameters of the components in a pivot bearing. The assumption is necessary to simplify the modeling of a pivot bearing when dimensional of the retainer is neglected and considered as one-dimensional. Thus the model can be generated loop analysis of a pivot ball bearing is shown in Figure 4.

Figure 4. Loop analysis of a pivot ball bearing.

where

\[ A = \text{The curvature of the outer raceway to the outer race diameter.} \]
\[ B = \text{The outer race diameter to the center line of pivot.} \]
\[ C = \text{The center line of pivot to the inner diameter of the inner race.} \]
\[ D = \text{The inner diameter of the inner race to the curvature of the inner raceway.} \]
\[ E = \text{The curvature of the inner raceway to a starting point of gap. (Ball diameter)} \]

All of parameters are generated by loop analysis can be described by the equation (6)

\[ \overline{\text{Gap}} + A - B - C - D - E \]  \hspace{1cm} (6)

From equation (6) can be written as

\[ \overline{\text{Gap}} = B - A - C - D - E \]  \hspace{1cm} (7)

4.3 A list concerned parameters, the conversion of dimensions to bilateral tolerance forms and the specified constants for geometric, probability, and statistical multiplier, which is shown in Table 1
Table 1. Min/max tolerance charts for parts in Figure 4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nom</th>
<th>Nom T</th>
<th>Min T</th>
<th>Max T</th>
<th>Nom Val</th>
<th>Min Val</th>
<th>Max Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>-0.268</td>
<td>0.000</td>
<td>0.000</td>
<td>1</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>B</td>
<td>4.705</td>
<td>1.4705</td>
<td>0.000</td>
<td>0.000</td>
<td>1</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>C</td>
<td>3.170</td>
<td>1.3170</td>
<td>0.000</td>
<td>0.000</td>
<td>1</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>D</td>
<td>0.209</td>
<td>1.0209</td>
<td>0.000</td>
<td>0.000</td>
<td>1</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>E</td>
<td>0.100</td>
<td>1.0100</td>
<td>0.000</td>
<td>0.000</td>
<td>1</td>
<td>1</td>
<td>0.000</td>
</tr>
</tbody>
</table>

From Table 1 can be calculated the results below:

Worst case analysis case:
- Nominal gap = 0.115 mm.
- Maximum gap = 0.115 + 0.029 = 0.144 mm.
- Minimum gap = 0.115 - 0.029 = 0.086 mm.

Statistical analysis case:
- Nominal gap = 0.115 mm.
- Maximum gap = 0.115 + 0.018 = 0.133 mm.
- Minimum gap = 0.115 - 0.018 = 0.096 mm.

Regression Analysis: Gap versus B, C, E

The regression equation is:

$$\text{Gap} = -0.068 + 1.90 \times B - 1.09 \times C - 1.09 \times E$$

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.268</td>
<td>0.202</td>
<td>-2.31</td>
<td>0.090</td>
</tr>
<tr>
<td>B</td>
<td>0.909</td>
<td>0.202</td>
<td>4.50</td>
<td>0.000</td>
</tr>
<tr>
<td>C</td>
<td>-1.209</td>
<td>0.202</td>
<td>-5.98</td>
<td>0.000</td>
</tr>
<tr>
<td>E</td>
<td>-0.599</td>
<td>0.202</td>
<td>-2.96</td>
<td>0.090</td>
</tr>
</tbody>
</table>

$$R^2 = 0.602526 \quad R^2_{(adj)} = 0.664$$

Analysis of Variance

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>3.642</td>
<td>1.214</td>
<td>1.41</td>
<td>0.006</td>
</tr>
<tr>
<td>Residual Error</td>
<td>9.909</td>
<td>0.900</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>9.999</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source</th>
<th>Deg df</th>
<th>SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>1</td>
<td>0.621</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>0.387</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>0.293</td>
</tr>
</tbody>
</table>

Figure 5 shows the plotted results of different gap values.
Therefore, the mathematical modeling equation can be generated by linear regression analysis as 
\[
\text{Gap} = -0.588 + 1.00B - 1.00C - 1.00E \quad (8)
\]
where 
\begin{align*}
\text{Gap} & = \text{Gap values.} \\
B & = \text{The outer race diameter to the center line of a pivot.} \\
C & = \text{The center line of a pivot to the inner diameter of the inner race.} \\
E & = \text{The curvature of the inner raceway to a starting point of gap. (Ball diameter)}
\end{align*}

From the equation (8), the calculated results are shown below:

Mathematical modeling equation:
- Nominal gap = 0.116 mm.
- Maximum gap = 0.130 mm.
- Minimum gap = 0.102 mm.

**Comparison of results**

Table 2 presents the results of three methods which are the worst case analysis, statistical analysis, and the common tolerance analysis. Gap values between basic principles and mathematical modeling equation are shown in Table

**Table 2.** The results of basics principles and mathematical modeling equation.

<table>
<thead>
<tr>
<th>Types</th>
<th>Basic principles</th>
<th>Math modeling equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Worst case analysis</td>
<td>Statistical analysis</td>
</tr>
<tr>
<td>Max Gap</td>
<td>0.086 mm</td>
<td>0.096 mm</td>
</tr>
<tr>
<td>Nom Gap</td>
<td>0.115 mm</td>
<td>0.115 mm</td>
</tr>
<tr>
<td>Min Gap</td>
<td>0.144 mm</td>
<td>0.133 mm</td>
</tr>
</tbody>
</table>

**Figure 10.** Different gap values.

The gap values results of each method are slightly different. In actual work, the usage of each method depends on the user and the appropriateness of the work. The flexibility of this study allows the mathematical modeling equation to be generated in order to describe the relationship between individual parts.

**Conclusion**

This study is based on the worst case analysis and statistical analysis which provides the maximum clearance and minimum clearance as expressed in the Min/Max Chart. With the common tolerance analysis methods, Min/Max Charts were combined with Parametric Simulation to generate the mathematical modeling equation by linear regression analysis. The results of worst case analysis differs from mathematical modeling equation −0.87% in nominal gap, 9.72% in maximum gap, and −18.60% in minimum gap. In addition, the result of statistical analysis differs from mathematical modeling equation −0.87% in nominal gap, 2.26% in maximum gap and −6.25% in minimum gap.
Acknowledgment

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